Problem 10.5

Kinematics was predicated on the fact that if you have a body moving under the influence of a constant acceleration, there will be relationships between the body's *position*, *speed* and *acceleration* over a given *time interval* that will ALWAYS BE TRUE. So you have, for instance, the relationship:

$$x_2 = x_1 + v_{1,x}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2$$

What does this tell you? It says that if you want to calculate a body's x-coordinate at some "second point-in-time" x_2, you have to start with its coordinate at some initial point-in-time x_1, and to that add:

--the position-change over the time interval Δt due to the fact that the body had some initial velocity $v_{1,x}$, and

$$\Delta x_{dueToInitialVelocity} = v_{1,x}(\Delta t)$$

--the position-change over the time interval Δt due to the fact that the body was accelerating:

$$\Delta x_{dueToAcceleration} = \left(\tfrac{1}{2}\right)a_x(\Delta t)^2$$

Execute all that and you will end up with the new coordinate.

The beauty of all of this is that there is a rotational counterpart to the kinematic equations, and those equations have the exact same form as do the translational ones, only with rotational parameters replacing the translational parameters. The main ones are listed below.

$$v_{2,y} = v_{1,y} + a_y(\Delta t) \quad \Rightarrow \quad \omega_2 = \omega_1 + \alpha(\Delta t)$$

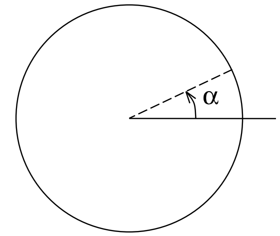
$$x_2 = x_1 + v_{1,x}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2 \quad \Rightarrow \quad \theta_2 = \theta_1 + \omega_1(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$$

$$(v_{x,2})^2 = (v_{x,1})^2 + 2a_x\Delta x \quad \Rightarrow \quad (\omega_2)^2 = (\omega_1)^2 + 2\alpha\Delta\theta$$

Note: The rotational parameters have "1's" and "2's" subscripts, denoting *point in time 1* and *point in time 2*, but there are no "x" or "y" subscripts. This is because in translational kinematics, you need the option of dealing with a two dimensional situations. In rotational motion, if the *line of the axis* about which the rotation happens *does not change* with time, you are looking at a *one dimensional* situation. As that is the only kind of situation you will run into in this class, you will never include "x" or "y" type information in rotational problems. (More will be said in class how rotational parameters are made into vectors. For now, just get used to using the equations.)

So to the problem:

A wheel angularly accelerates from rest to 12.0 rad/s in 3.00 seconds.



a.) What is the wheel's angular acceleration?

As is the case with most kinematics problems, this is a situation in which you have to identify what you are given, identify what you want, then find a kinematic equation that has it all. In this case, you know:

$$\theta_1 = 0,\ \omega_1 = 0,\ \omega_2 = 12 \text{ rad/sec},\ \text{and } \Delta t = 3.00 \text{ sec}.$$

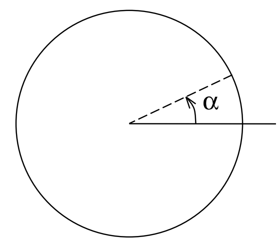
(Writing it out is always good!)

We want α. The relationship that does the job is:

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{(12.0 \text{ rad/sec})}{(3.00 \text{ sec})} = 4.00 \text{ rad/sec}^2$$

b.) What is the wheel's angular displacement?

The known information now is:



$$\theta_1 = 0,\ \omega_1 = 0,\ \omega_2 = 12 \text{ rad/sec},$$
$$\alpha = 4.00 \text{ rad/sec}^2 \text{ and } \Delta t = 3.00 \text{ sec}.$$

The equation that does the job this time is:

$$\theta_2 = \theta_1 + \omega_1(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$$

$$\Rightarrow \quad \theta_2 - \theta_1 = \omega_1(\Delta t) + \frac{1}{2}\alpha(\Delta t)^2$$

$$\Rightarrow \quad \Delta\theta = \overset{0}{\cancel{\omega_1(\Delta t)}} + \frac{1}{2}\alpha(\Delta t)^2$$

$$= \frac{1}{2}(4.00 \text{ rad/sec}^2)(3.00 \text{ sec})^2$$

$$= 18.0 \text{ radians}$$